Méthode des forces

1. Degré d'hyperstaticité :

$$d = 3c - a - 2s$$

2. Equations Canoniques :

$$[\delta_{ij}]\{X_i\} + [\delta_{io}] = 0.$$

δ_{ij} : coefficient de flexibilité (déplacement produit dans la section i causée par une force $X_j = 1$).

$$\delta_{ij} = \delta_{ji} = \frac{1}{EI}\int_0^\ell m_j m_i \, dx$$

$$\delta_{io} = \frac{1}{EI}\int_0^\ell M_o \, m_i \, dx$$

$$\delta_{ii} = \frac{1}{EI}\int_0^\ell m_i m_i \, dx$$

- هذه الطريقة تحسن استخدامها في الحالات التالية :

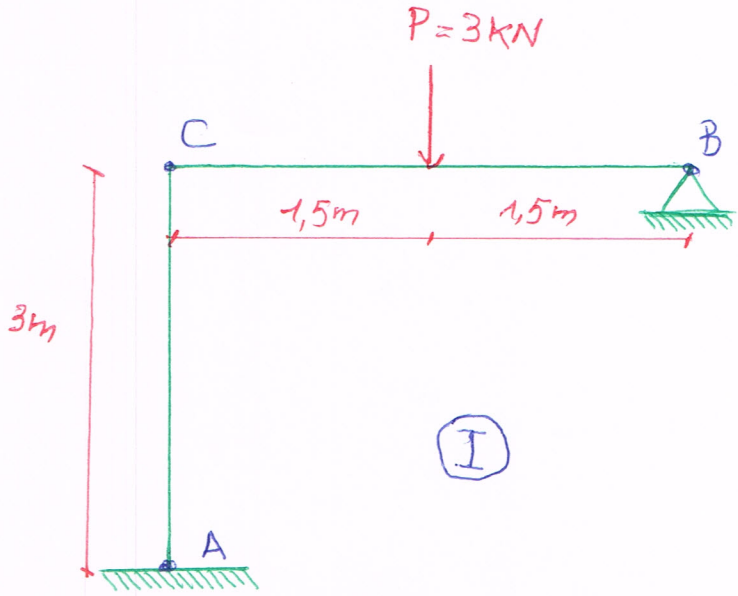
  - Poutres
  - Portiques
  - Système en treillis

- وطريقتها توفر معادلات إضافية.

Exo1 :

EI constante

Tracer les Diagrammes M, N, T.

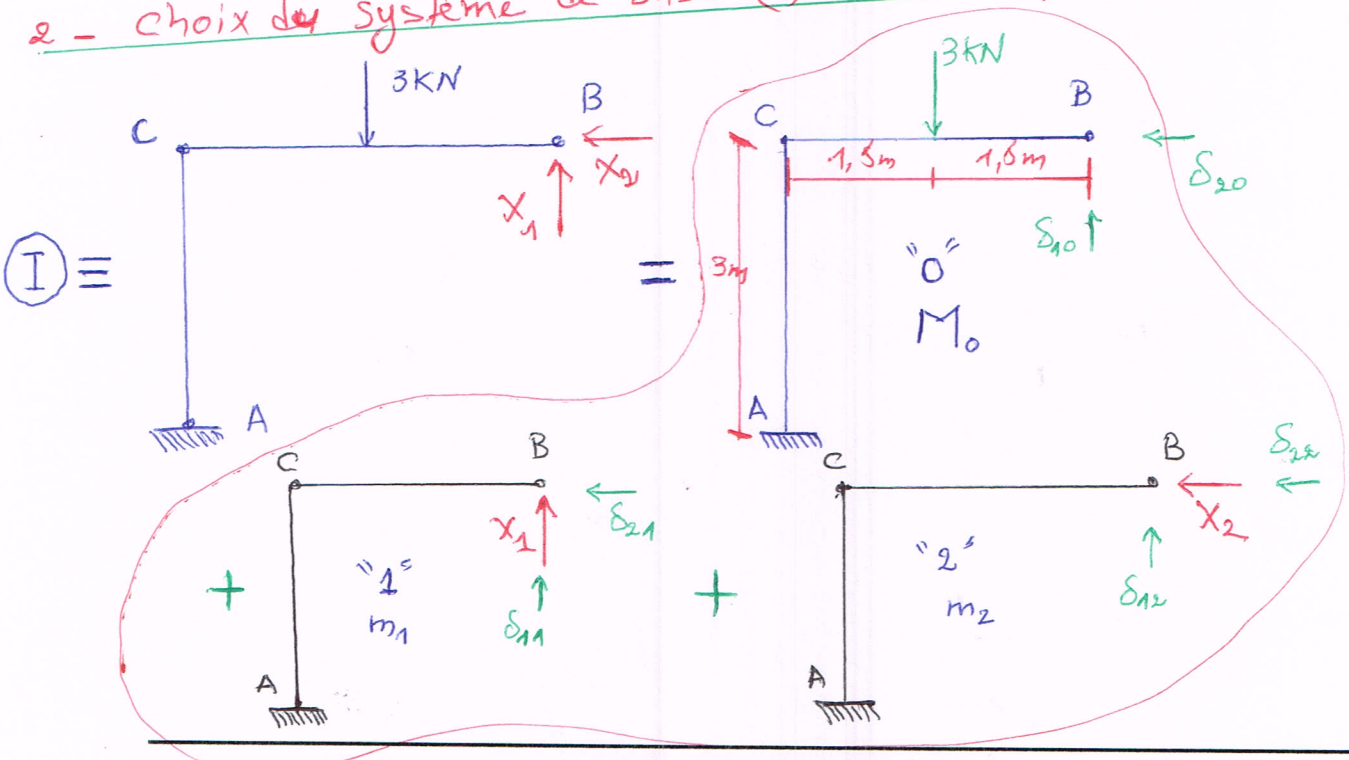


Solution :

1 - $d = 3c - a - 2S$ (Degré d'hyperstatisité)

$d = 3(1) - 1 - 2(0) = 2$

2 - choix du système de base (fondamental)



3- les équations canoniques :

$$\delta_{V}^{B} = 0 \Leftrightarrow \delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0$$
(appui)

$$\delta_{H}^{B} = 0 \Leftrightarrow \delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0.$$
(appuis)

$$\Leftrightarrow \begin{cases} \delta_{11} x_1 + \delta_{12} x_2 + \delta_{10} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{20} = 0 \end{cases}$$

4- Calcul des coeff. de Flexibilité δ_{ij} :

1) $\delta_{11} = \frac{1}{EI} \int m_1 m_1 \, dx$ 2) $\delta_{22} = \frac{1}{EI} \int m_2 m_2 \, dx$

3) $\delta_{21} = \delta_{12} = \frac{1}{EI} \int m_1 m_2 \, dx$

4) $\delta_{10} = \frac{1}{EI} \int M_0 m_1 \, dx$

5) $\delta_{20} = \frac{1}{EI} \int M_0 m_2 \, dx$

$$\delta_{11} = \frac{36}{EI} \qquad \delta_{12} = \delta_{21} = \frac{13,5}{EI} \qquad \delta_{22} = \frac{9}{EI}$$

$$\delta_{10} = -\frac{48,94}{EI} \qquad \delta_{20} = -\frac{20,25}{EI}$$

$$\begin{cases} \frac{36}{EI} x_1 + \frac{13,5}{EI} X_2 - \frac{48,94}{EI} = 0 \ldots\ldots (1) \\ \frac{13,5}{EI} x_1 + \frac{9}{EI} X_2 - \frac{20,25}{EI} = 0 \ldots\ldots (2) \end{cases} \Leftrightarrow \begin{cases} X_1 = 1,19 \text{ KN} \\ X_2 = 0,48 \text{ KN} \end{cases}$$

5. Corrections des diagramme "1" et "2" ds moments :

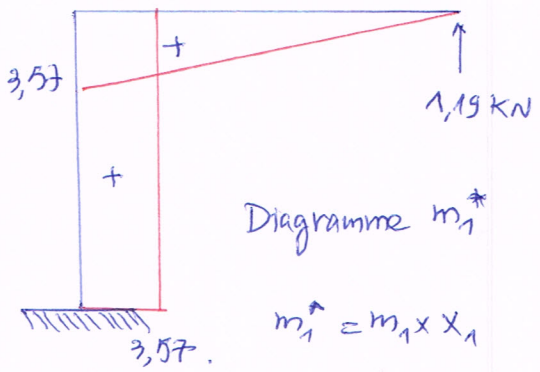


Diagramme m_1^*

$m_1^* = m_1 \times X_1$

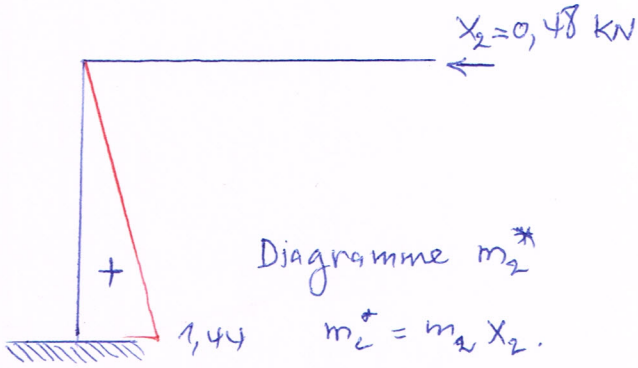
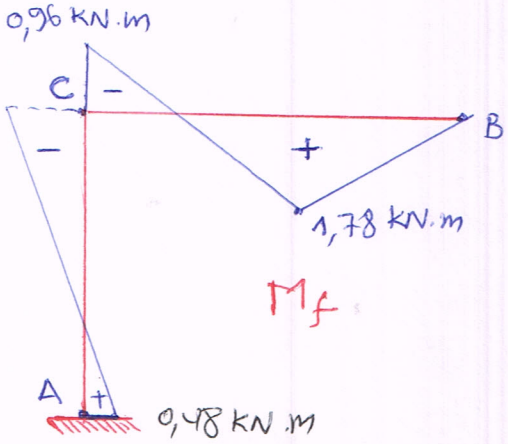


Diagramme m_2^*

$m_2^* = m_2 X_2$.

6. Diagramme final :

$$M_f = M_0 + m_1^* + m_2^*$$



M_f

← يمكن استخدام الطريقة السابقة لحساب X_1 و X_2 مباشرة من أجل رسم $M(x)$ $T(x)$ $N(x)$.

المرجع :

→ Mohammed MEKKi, "Calcul des structures hyperstatiques : Cours et exercices corrigés" Polycopié, Univ. d'Oran – 2016/2017 –